BASEL LANDSCHAFT
BILDUNGS-, KULTUR- UND SPORTDIREKTION
GYMNASIUM OBERWIL

Matura Exam 2022
Mathematics - Grundlagenfach

Class:	4b / 4c / 4d / 4e / 4f / 4g / 4h / 4k
Number of pages: (without cover page)	6
Contents:	Matura exam 2022, Mathematics written, Grundlagenfach Problem 1: Short problems, 14 points Problem 2: Stochastics, 13 points Problem 3: Vector geometry, 14 points Problem 4: Calculus, 6 points Problem 5: Calculus, 15 points Problem 6: Calculus, 12 points
Instructions/ explanations:	Use a **separate sheet for each problem**.
Auxiliaries:	Formulary Mathematics kompact (German or English) (Adrian Wetzel / ISBN: 978-3-9523907-5-7) Calculator Ti-30X Pro MultiView or Ti-30X Pro MathPrint Dictionary English - German
Grading:	Maximum 74 points. The maximum points are indicated with each problem. For grade 6 the maximum points are not required.

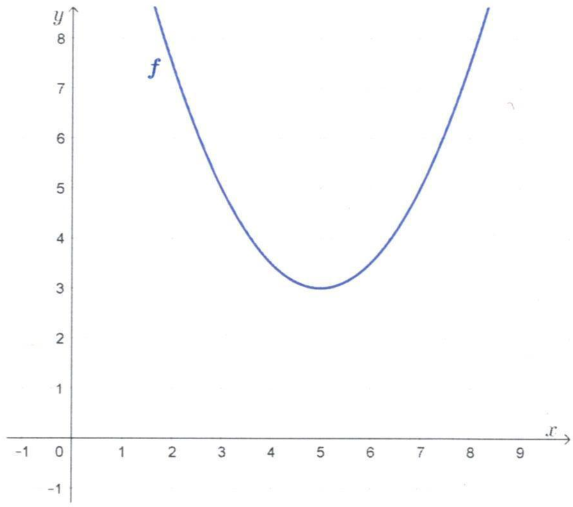
Before you start solving the problems, please check whether the exam is complete according to the listing above. In case you do suspect something missing, please report to the supervisors **immediately**.

Problem 1 | 14 points

(1.5P+2P+2.5P+2P+2P+2P+2P)

a) Determine a function equation of the function f.

(Read suitable points from the figure)



b) Calculate the exact volume of the solid of revolution which is created by revolution of the graph of $f(x) = x^4$ about the x-axis on the interval $I = [0; 3]$.

c) Given are the points $A(1|4|5)$, $B(2|3|6)$ and $C(3|3|5)$. Determine a Cartesian equation of the plane through the three points.

d) Given is the plane $E: 3x - 4y + z = 5$ and the point $P(10|-11|9)$. The line g runs perpendicularly to the plane through P.
Calculate the intersection of E and g.

e) Given are the points $A(1|2|4)$ and $B(5|2|8)$. Determine an equation of the plane which comprises all points with equal distance from A and B.

f) In an urn we have 6 balls, 4 of which are black and 2 are white. From these, 2 balls are drawn without returning. The random variable X indicates the number of black balls drawn. Calculate the expected value $E(X)$.

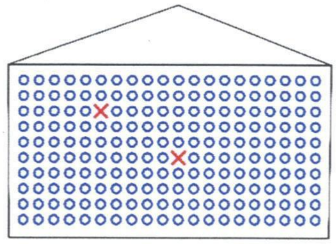
g) Determine all numbers which increase by 2 when squared.

Problem 2 | 13 points

(1P+1P+3P+2P+2P+2P+2P)

The hotel Bernoulli has 200 rooms, which all can be booked independently from each other.

a) 198 rooms are booked. You can see the «key rack» of the hotel. 200 rooms out of 198 are booked.
How many ways can the key rack look like for the 198 booked rooms?



The Hotel Bernoulli has four different kinds of rooms (see Table 1).

b) At a special weekend there is a unit price of CHF 99.– for each room. All rooms are raffled randomly.
What is the probability that a deluxe room is drawn as the first?

room category	number	p… /night
budget room	80	…0. –
standard room	60	CH…
comfort room	40	CHF …
deluxe room	20	CHF 50…

Special 99.-

Table 1

c) A group of three has booked a room each at this special weekend.
What is the probability that …
i) … each of the three is alloted a budget room?
ii) … exactly two of the three is alloted a standard room?

d) On another (normal) weekend all rooms of the hotel have been booked (see Table 2). From past experience it is known that the probability for guests showing up when they have a booking is 90%. What income can the hotel expect at this weekend if only guests showing up pay?

room category	number	price/night
budget room	80	CHF 100. –
standard room	60	CHF 200. –
comfort room	40	CHF 300. –
deluxe room	20	CHF 500. –

Table 2

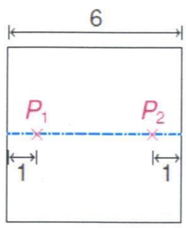
For the following calculations also assume a probability of 90% of guests showing up.

e) What is the probability that, of 20 randomly picked bookings, exactly three do not show up.

f) What is the maximum number of bookings that the hotel can accept so that the probability for the hotel to be overbooked is less than 1%?

g) In the peak season (265 days), exactly 220 bookings are accepted for each night. This means that sometimes guests have to be turned away despite a valid booking. On how many days is this to be reckoned with?

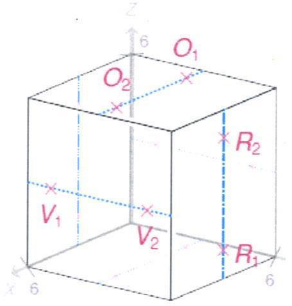
Problem 3 | 14 points (1P+3P+2P+2P+2P+4P)



We consider a cube with edge length 6. One vertex is at the origin, three edges lie on the coordinate axes.

(1) On each face of the cube *one* of the center lines is drawn.

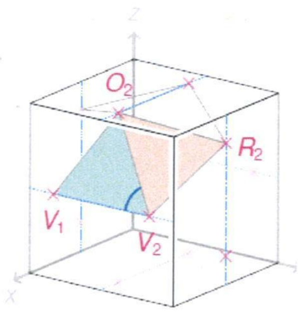
1



2

(2) On each of these center lines – at a distance of 1 unit from the respective end – two points are drawn.

(3) Connecting three of these points respectively creates a solid **(4)** in this cube.



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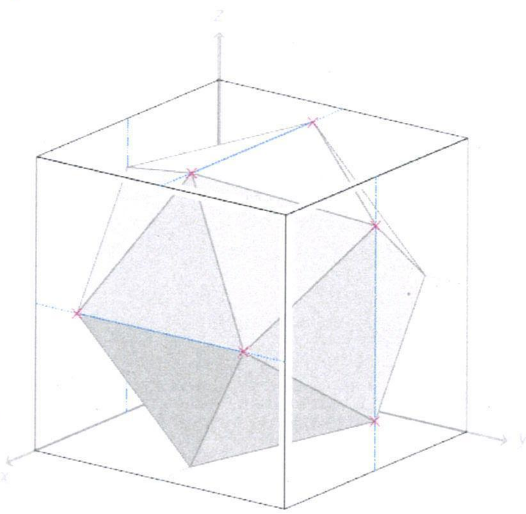


4

a) Determine the coordinates of the **point** V_2.

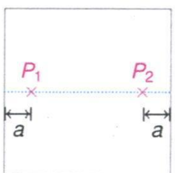
b) Calculate the **angle** $\sphericalangle V_1V_2O_2$ (vertex in point V_2).

c) Show, that the **triangle** $V_2R_2O_2$ has three equally long sides.

d) How many triangles make up the surface of the entire solid?
How many of these triangles are equi**lateral**, how many are not?
Indicate the way you have been counting!

e) The solid is intersected with the horizontal plane $z = 3$.
How does the sectional figure look like?
Create an expressive **sketch** of the sectional figure and calculate the **size of the area** of the sectional figure.

f) We will now move the points on the respective center line.



Thereby all triangles can be made equilateral.
Determine the value of a so that the triangle $V_1\ V_2\ O_2$ gets three **equally long** sides.

Problem 4 | 6 points

(1P+2P+2P+1P)

Aphids* multiply on a houseplant. The population of aphids $P(t)$ changes with the rate of change

$$P'(t) = -0.06t^2 + 2.4t$$

where t is the time in weeks ($t \geq 0$) and $P'(t)$ the change in the population in number of aphids per week.

a) Calculate the zeros of $P'(t)$.

b) Calculate the time when the aphid population **increases the most**.

c) Find the function equation of the function $P(t)$, which describes the number of aphids as a function of time. Assume, that at the start of the measurement ($t = 0$ weeks) there are 10 aphids on the houseplant.

d) Calculate the maximum number of aphids on the houseplant that is reached according to this mathematical model.

* aphid: Blattlaus

Problem 5 | 15 points

(6P+2P+3P+1.5P+2.5P)

Given is the function f by

$$f(x) = 5x \cdot e^{-x+1}$$

a) Calculate the zeros and the exact coordinates of the extrema and inflection points. The proof that it is really an inflection point is not required.

b) Sketch the graph of the function f on the interval [0;10]. Use the points from a) plus at least three more reasonably chosen points.

c) Calculate the line equation of the tangent at $x = 4$ **exactly**.

d) Show, that the function F with

$$F(x) = (-5x - 5) \cdot e^{-x+1}$$

is an antiderivative of the function f.

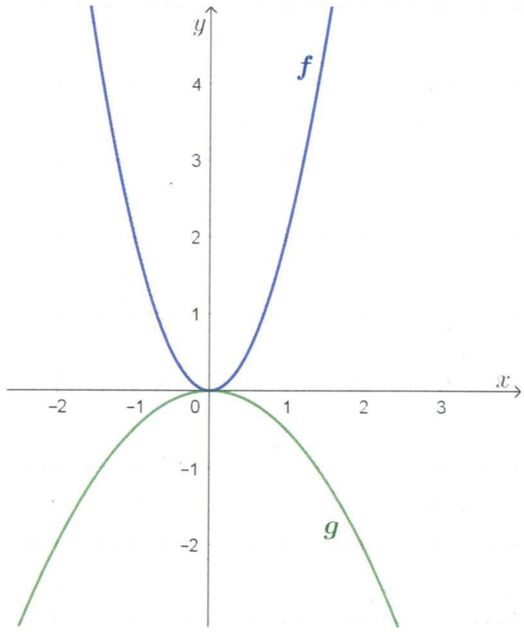
e) Calculate the size of the area open to the right that is enclosed by the graph of f and the x-axis in the 1st quadrant.

Problem 6 | 12 points

(6P+6P)

Given are the two functions f and g (see figure).

$$f(x) = 2x^2 \qquad g(x) = -\frac{1}{2}x^2$$



a) Between the function graphs of the functions f and g a triangle with the vertices $A(x|g(x))$, $B(2|0)$ and $C(x|f(x))$ is inscribed.

Set up a term for the described triangular area.

For what value of x between 0 and 2 does the size of this area become maximal?

b) By introducing the variable a the functions f and g are now generalized to become families of functions f_a and g_a:

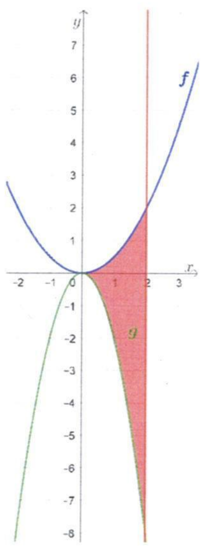
$$f_a(x) = ax^2 \quad g_a(x) = -\frac{1}{a}x^2 \quad \text{where } a > 0$$

For f_a as well as for g_a the same value for a is chosen.

An area is bounded by the graph of f_a, the line $x = 2$ and the graph of the function g_a in the 1st and 4th quadrant. As an example below you can see two figures for two different values of a.
Show that this area will be minimal for $a = 1$. Checking the interval bounds is not required.

Calculate the size of the area for this value of a.

$a = 0.5$



$a = 2$