BASEL
LANDSCHAFT

BILDUNGS-, KULTUR- UND SPORTDIREKTION
GYMNASIUM OBERWIL

Final Exam 2021
Mathematics - Basic Course / Immersion

Class / Course:	4b
Number of pages (not counting cover):	8
Content:	Written final exam 2021 in mathematics, basic course, comprising 8 exam questions
Instructions:	Please start every question at the top of a **new page**.
Resources:	Scientific calculator **without** graphing capabilities TI-30X Pro MultiView Formelsammlung Mathematik **kompakt** (German or English) (Adrian Wetzel / ISBN 978-3-9523907-5-7)
Scale:	The total is **61** points The number of points is stated for each question. You do not need to attain the full number of points for mark 6.

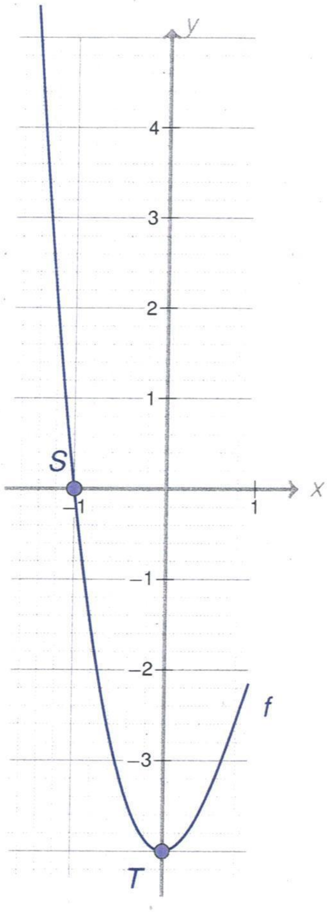
Before you get started solving the questions, please make sure your exam paper is complete and matches the above description. If you think that something is missing, please report to the supervisors **immediately**.

Question 1:

4+3.5=**7.5** points

The following graph represents a polynomial function *f* with

$$f(x) = -x^3 + bx^2 + cx + d$$



Points *S* and *T* have integer coordinates, the values of which can be read from the figure.
T is a low-point of the graph.

a) Determine the function equation of *f*, based on the given points.

(In case you were unable to solve ***a)****, continue with the alternative function* $f(x) = -x^3 + 6x^2 - 32$.*)*

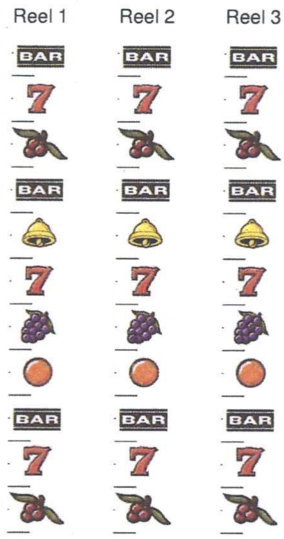
b) Determine the point of inflection and the coordinates of the missing high- or low-point of *f*. Specify whether that point is a high-point or a low-point.

Question 2:

1.5+1.5+1+2+2=**8** points

A slot machine (also known as one-armed bandit) is a gambling machine with three reels ("Walzen") with symbols on them. The reels spin and then stop in a random position. There are several winning combinations — we will only consider the **main prize** for which **each reel must display the symbol "7"**.

The three reels are identical. Each has 11 positions and bears the following symbols:



a) How many possible combinations of symbols are there which **look different**?

b) How many possible combinations of symbols are there which **look different** and involve **three different symbols**?

c) What is the probability that all three reels stop at a "7"?

d) What is the probability of getting at least one "BAR" symbol?

So far, all calculations have been based on the assumption that each of the 11 symbols is equally likely to be displayed. This assumption is known to be wrong:

A halting mechanism (US Patent 4,448,419 from 1984) covertly changes the behaviour of the reels — each reel will completely skip two of its "7" symbols, and it will stop at the third "7" symbol with a probability of only $\frac{1}{32}$.

e) Unsuspecting players believe that they win with the probability calculated in **c)**.
In reality, their chances are much worse. Calculate **by what factor** the chances have decreased.

Question 3:

3.5+2.5=**6** points

Consider the straight line g: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad (t \in \mathbb{R})$,

and the points $A(5|-2|13)$ and $C(17|2|10)$.

a) The triangle ABC is right-angled with its right angle at corner A.

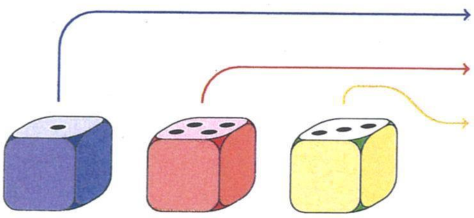
B lies on the line g.

Calculate the coordinates of B.

b) The point $P(5|6|17)$ lies on the line g as well.

A regular die is rolled 3 times; its scores determine a vector $\vec{v}$.

Example:



$$\begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$$

After rolling the die, we displace the point P by the vector $\vec{v}$.

Calculate the probability that the displaced point, P', is still on the line g.

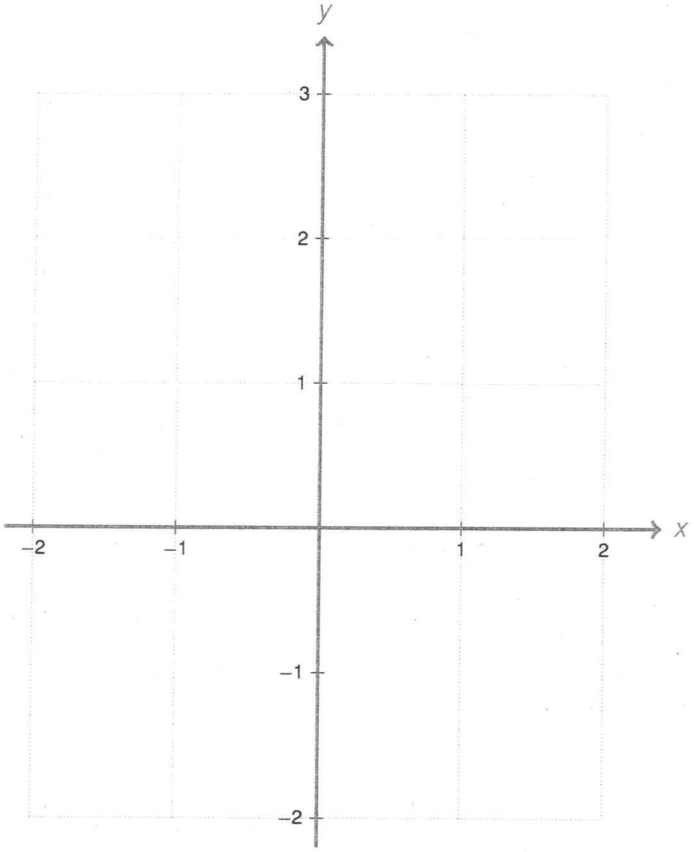
Question 4:

2+2.5+3=**7.5** points

Consider the function f, defined as $f(x) = e^{-x}$. Also, $g: y = x + 1$ is a straight line.

a) Draw the graph of f and the line g in the coordinate system on this sheet.

▷*As an alternative, you may also draw the graph and the line on your answer sheet. Please use the same scale.*



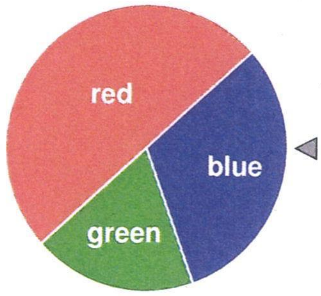
b) Show by calculation that both the line g and the graph of f pass through the point $P(0|1)$, and that they intersect at a right angle.

c) The x axis, the line g and the graph of f enclose a region which stretches indefinitely to the right. The y axis divides this region into two parts. What is the **ratio** between the areas of these two parts?

Question 5:

3+2+1.5=**6.5** points

The figure shows a wheel of fortune with three sectors:
The **red** sector covers **half** the wheel,
hence $P(\text{red}) = \frac{1}{2}$.



The probabilities of the other outcomes, $P(\text{green})$ and $P(\text{blue})$, are not yet known.

What we do know is the probability of the event E "twice the same colour if the wheel is spun twice":

$$P(E) = \frac{7}{18}$$

a) Calculate the probabilities $P(\text{green})$ and $P(\text{blue})$ if you spin the wheel once.

▷*Note: the figure is not drawn to scale; nonetheless, the blue sector is larger than the green sector.*

*(In case you were unable to solve **a)**, use* $P(\text{red}) = \frac{1}{2}$, $P(\text{green}) = \frac{1}{8}$, $P(\text{blue}) = \frac{3}{8}$ *to continue.)*

b) Someone invites you to a game involving this wheel of fortune. You have to place a bet of CHF 3.– for each round (in which you get to spin the wheel once).

The game has the following rules:

red: you lose your money.
blue: you get your money back, plus another CHF 1.–
green: you get your money back, plus another CHF 3.–

Would you play this game? (Give reasons for your answer)

c) You spin the wheel of fortune 100 times. What is the probability that it stops in the green sector at least 20 times?

Question 6:

2+4.5=**6.5** points

A parabola p with equation $p(x) = -x^2 + 8$ is given, along with the two fixed points $A(-1|0)$ and $B(1|0)$.

Points C and D are located on the parabola p, in such a way that the quadrilateral $ABCD$ is a trapezium, symmetric with respect to the y axis (see figure).

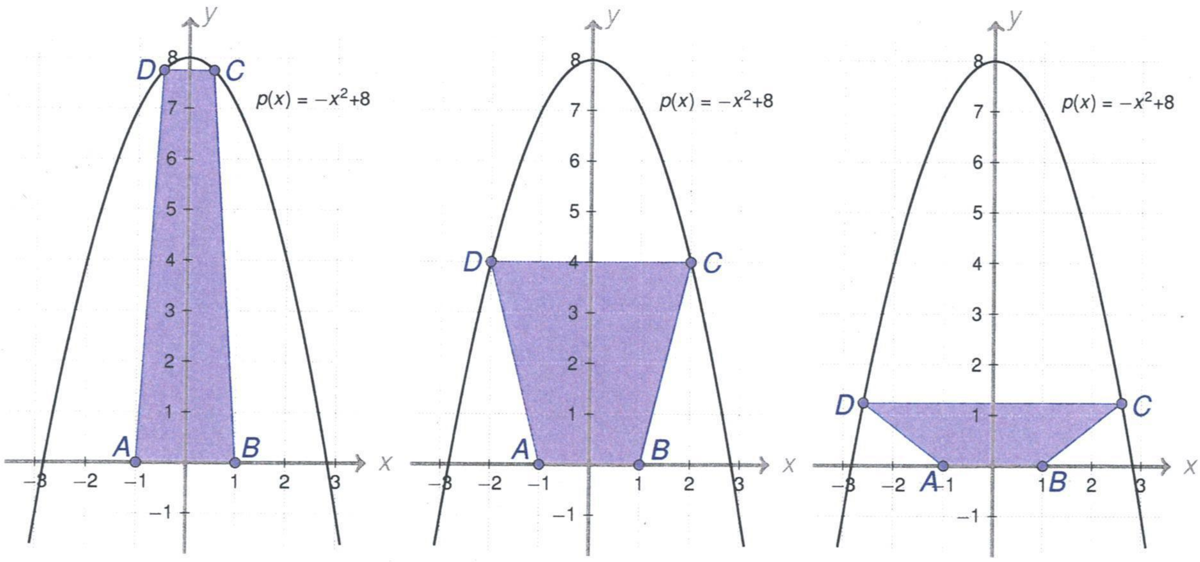


Figure: Three versions of the trapezium, depending on the position of C and D.

a) Calculate the area of the trapezium $ABCD$ for $C(2|4)$ and $D(-2|4)$ (cf. middle figure).

b) The points C and D are allowed to slide along the parabola, such that $ABCD$ remains a trapezium above the x axis and is still symmetric with respect to the y axis.
Calculate the maximum possible area of the trapezium $ABCD$.

Question 7: 2+3+2+4=**11** points

Given the point M with

$$M(3|-6|6),$$

the straight line g with

$$g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 4 \end{pmatrix} + s \cdot \begin{pmatrix} 5 \\ 1 \\ -2 \end{pmatrix} \quad (s \in \mathbb{R})$$

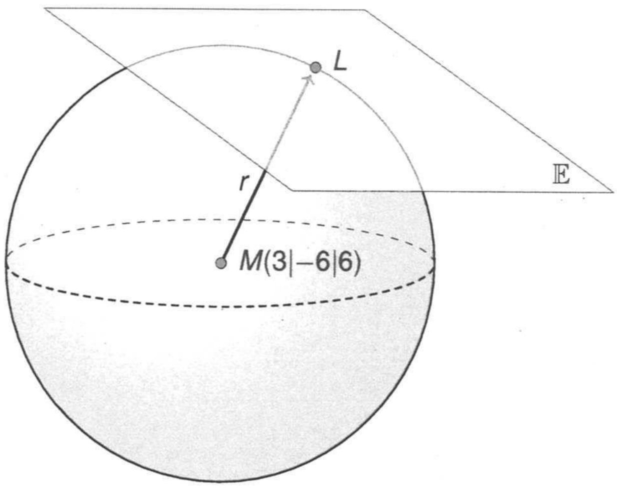
and the plane $\mathbb{E}$ with

$$\mathbb{E}: 2x - 2y + z - 6 = 0.$$

a) The line g intersects the plane $\mathbb{E}$. Calculate the **angle** of intersection.

b) Calculate the distance r between point M and plane $\mathbb{E}$, as well as the perpendicular foot L.

Let M be the centre of a sphere $\mathcal{K}$ and L a point on the surface of the sphere.



▷ *The following parts can be solved without* ***a)*** *and* ***b)*** *if you assume $r = 6$ for the radius of the sphere.*

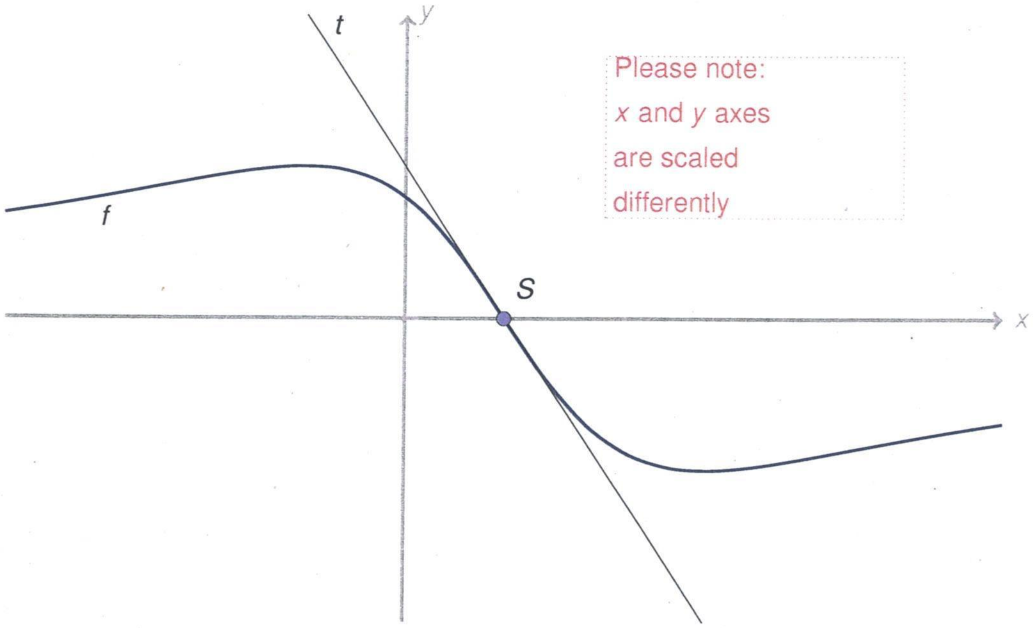
c) What point P on the surface of the sphere is closest to the origin $O(0|0|0)$?

d) The line g intersects the sphere $\mathcal{K}$ at two points, G and H.
Calculate the coordinates of these points.

Question 8:

1+3+2+2=**8** points

Consider the function $y = f(x) = \dfrac{1-x}{x^2-2x+5}$ which is defined for all $x \in \mathbb{R}$ (see sketch):



a) Determine the point of intersection S between the graph of f and the x axis.

b) Calculate the equation of the tangent t touching the graph of f at S.

c) Show that $F(x) = -\frac{1}{2} \cdot \ln(x^2 - 2x + 5)$ is an antiderivative of $f(x)$.

d) Calculate the area of the finite region defined by the graph of f and the positive x and y axes. The result should be an **exact** value.