BASEL LANDSCHAFT
BILDUNGS-, KULTUR- UND SPORTDIREKTION
GYMNASIUM OBERWIL

Final Exam 2019
Mathematics - Basic Course / Immersion

Class / Course: 4a_glf / 4b_glf / 4c / 4d / 4e/ 4f / 4g / 4h / 4i

Number of pages (not counting cover): 4

Content: Written final exam 2019 in mathematics, basic course, comprising 6 exam questions

Instructions: Please start every question at the top of a **new page.**

Resources: Formelsammlung Mathematik **kompakt** (German or English)
(Adrian Wetzel / ISBN 978-3-9523907-5-7)

Calculator
TI82Stats, TI83, TI83+, TI84+, TI84+ Silver Edition, TI84+ CE-T

Scale: The total is 79 points

The number of points is stated for each question.
You do not need to attain the full number of points for mark 6

Hiermit bestätige ich anhand des mir vorgelegenen Exemplars, dass die Prüfung korrekt und mit allen Unterlagen versehen, ausgefertigt ist.

6. Mai 2019

Datum, Unterschrift (AD)

Before you get started solving the questions, please make sure your exam paper is complete and matches the above description. If you think that something is missing, please report to the supervisors **immediately**.

Problem 1:

3+1+3+2+4= **13** points

A cuboid *ABCDEFGH* with the square *ABCD* as its base stands on the *xy*-plane. We know its vertices *B*, *C* and *G* and the midpoint *M* of the bottom square *ABCD*:

$$B(7|5|0) \qquad C(4|8|0) \qquad M(4|5|0) \qquad G(4|8|10)$$

a) Determine the vertices *A*, *D* and *E*.

b) Calculate the length of the edges of the bottom square *ABCD*.

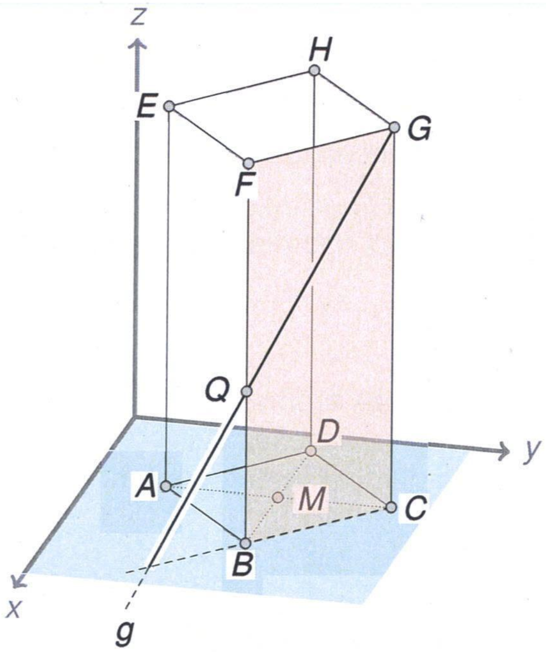
c) A line *g* lies in the side face *BCGF*. Its directional vector is $\vec{v}$.

$$\vec{v} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$$

In which point *Q* of the edge $\overline{BF}$ does the line *g* leave the cuboid's side face?

d) Calculate the angle of inclination φ of the line *g* with respect to the *xy*-plane.

e) A normal line to the line *g* passes through the point *E*. Calculate its point of intersection L_g with the line *g*, and the distance between the point *E* and the line *g*.



Problem 2:

7+2+3+3= **15** points

Given is the function *f* with

$$y = f(x) = \frac{4x-4}{x^2} = \frac{4}{x} - \frac{4}{x^2}$$

a) Determine its domain and calculate the point of intersection of the graph of *f* and the *x*-axis, possible maximum and minimum points and the point of inflexion of the function *f*.
▷ *You don't have to show that it's really a point of inflexion.*

b) Find the vertical and the horizontal asymptote of the graph of *f*.

c) Calculate the equation of the tangent *t* to the graph of *f* at $x = \frac{3}{2}$.

d) The horizontal line passing through the maximum of *f*, the graph of the function *f*, the *x*-axis and the *y*-axis border a finite region *A* lying in the first quadrant.
Draw a sketch, mark the described region *A* and calculate its area (**exact** result).

Problem 3:

2+3+2+3+2+3= **15** points

Given is the triangle ABC with vertices $A(1|8|2)$, $B(7|-16|-4)$ and $C(1|-10|20)$.

a) Show that the triangle ABC is equilateral.

b) The point P complements the triangle ABC to a rhombus.
How many solutions are there for the position of P?
▷ *Give arguments for your answer using a sketch.*
Calculate the coordinates of **one** of the possible points P.

c) Calculate the center of gravity of the triangle ABC.

d) Calculate a Cartesian equation of the plane $\mathbb{E}$ passing through the points A, B and C.

e) Describe in words where all the points with the following properties lie:
Their distances to the vertices of the equilateral triangle ABC are equal.

f) The point Q has distance 18 from the points A, B and C.
Calculate its coordinates. ▷ ***One** solution is sufficient.*

Problem 4:

1+2+3+3+3= **12** points

A rumour spreads around the 900 students of the Gymnasium Oberwil. The spreading of the rumour is described using the following mathematical model: A function $K(t)$ denotes the number of persons who already know the rumour at the time t, where t is in days.

$$K(t) = \frac{900}{1 + 899 \cdot e^{-0.5 \cdot t}}$$

a) Calculate the number of persons who know the rumour after 5 days (i.e. at the time $t = 5$).

b) Calculate the time at which 600 persons know the rumour.

c) Sketch the graph of the function for the interval $0 \leq t \leq 36$.
▷ *You are allowed to use the graphing menu of your calculator for this task.*
Determine the value which $K(t)$ approximates for $t \to \infty$.
Explain in some words the meaning of this value.

d) Calculate the first derivative $K'(t)$.
Explain in some words the meaning of the derivative in this example.

e) The function $K(t)$ grows nearly exponentially at the beginning (i.e. in the time interval $0 \leq t \leq 10$).
Find a simpler function $N(t)$ of the form

$$N(t) = a \cdot e^{k \cdot t}$$

with the following properties:

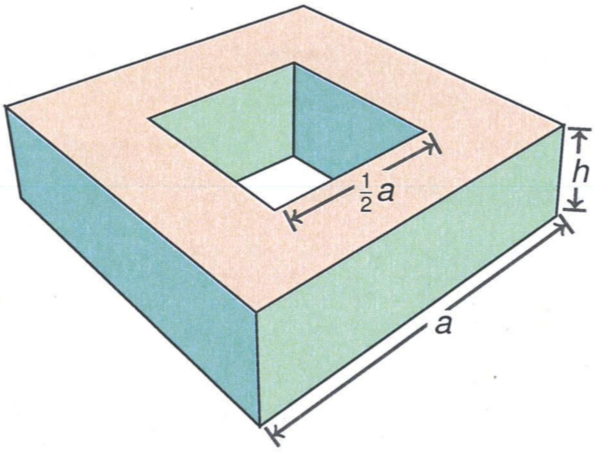
- At the time $\boldsymbol{t = 0}$ the values of $K(t)$ and $N(t)$ are equal.
- At the time $\boldsymbol{t = 0}$ the derivatives $K'(t)$ and $N'(t)$ are equal.

Determine the functon equation of $N(t)$.
*(In case you were unable to solve problem **d)**, continue your calculation with $K'(0) = 0.6$)*

Problem 5:

4+6= **10** points

A building in the shape of a cuboid has a square base and a square court on the inside. The volume of the building (without court) equals 2592 m^3.

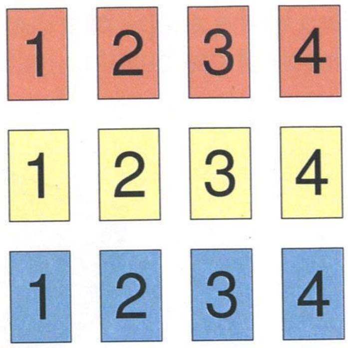


a) The architect decides, for the time being, that the side length a of the building should be 20 m. To be able to estimate the heat loss of the building he calculates the total surface of the building (not counting its bottom area). Calculate the height of the building and its surface.

b) To have as little heat loss as possible, the architect *rejects* his original decision regarding the side length and makes it his top priority to **minimize** the surface of the building (not counting its bottom area).
In this case, how should the width a and the height h of the cuboid be chosen? How large is the minimum surface? (The building's original volume of 2592 m^3 remains unchanged.)

Problem 6:

2+2+2+4+2+2= **14** points



Given are 12 cards numbered from 1 to 4. Every number occurs three times: once in red, yellow and blue.
The cards are shuffled. Albin draws three cards **without putting them back** and places them on the table one after the other.

Calculate the probabilities of the following events:

a) All three cards are the same colour.

b) At least one of the numbers is even.

c) The three numbers form an increasing sequence.
▷ *In other words: The second number is larger than the first, and the third number is larger than the second.*

Albin offers the following game to Bernd:

For a stake of 5 Swiss francs Bernd is allowed to draw cards until a repetition occurs, that means until a **colour** or **number** is drawn the second time. For every drawn card (including the last one) Bernd gets 2 Swiss francs.

d) What profit can Bernd expect in one game?

e) Bernd has decided to join the game. But he has bad luck: A repetition occurs with the second card already. Under these conditions, what is the probability that the **colour** was repeated?

Now, Bernd draws a card, **puts it back** and subsequently shuffles the card deck. This process is repeated 12 times.

f) What is the probability that he draws a 2 **at least** 5 times?