BASEL
LANDSCHAFT

BILDUNGS-, KULTUR- UND SPORTDIREKTION
GYMNASIUM OBERWIL

Final Exam 2018
Mathematics – Profile A

Classes:	4a/4bA	Mathematics teachers: Dr. Jonas Gloor, Christian Oehrli
Duration:	4 hours	
Number of pages: **(not counting this cover)**	5 pages	
Content:	7 problems	
Instructions:	Please start solving each question **at the top of a new page!**	
Resources:	Book of formulas (DMK/DPK/DCR – Begriffe, Formeln, Tabellen) Calculator (TI-83 or TI-84 with empty storage) The calculator regulations of the Gymnasium Oberwil apply English dictionary	
Scale:	Total number of points: 66 The number of points attainable in each question is stated. You do not need to attain the full number of points in order to have mark 6	

Please make sure you have got all documents listed above before starting to solve the questions. If you think that something is missing, inform the supervisor **immediately!**

OH

1. $\boxed{3+2+2+2+2 = 11 \text{ points}}$

Given are the points $A(-1, 2, 3)$, $B(-2, 3, 7)$ and $C(-3, 1, 5)$
and the plane ε: $2x-2y+z-6 = 0$.

a) Calculate the side lengths and the angles of the triangle ABC.
What special kind of triangle is it?

b) Show that the triangle ABC is parallel to the plane ε.

c) A line passes through the triangle's vertex C and is perpendicular to the plane ε.
Determine the perpendicular foot (Lotfusspunkt) F and the distance between C and the plane ε.

The points A, B and F are reflected about the point C:
A', B' and F' are the reflected points.
A solid with vertices A, B, F, A', B' and F' results.

d) Find the coordinates of the points A', B' and F'.

e) What is the name of the resulting solid? Give reasons for your answer.

2. $\boxed{4+4+2+3 = 13 \text{ points}}$

The function $y = f(x) = ax+b+\frac{c}{x}$ has the zero $x_1 = 1$ and

$H(2, 1)$ is a maximum point of its graph.

a) Determine a, b and c.

In case you are unable to solve a), continue your calculations with the function
$y = f(x) = -2x+10-\frac{8}{x}$

b) Determine the other zeros of the function f and the other maximum / minimum points of the graph of f. Draw the graph of f.

c) Determine the area of the region between the graph of f and the x-axis lying in the first quadrant.

d) The point $P(0, 0)$ does not lie on the graph of f.
Determine the equation of the tangent to the graph of f passing through the point P.

3. $4 + 3 + 2 = 9$ points

Consider the function f defined by $f(x) = x^2 \cdot e^{-x}$.

a) Determine all zeros and maximum / minimum points of f.

b) Using **integration by parts** twice, show that $F(x) = -(x^2 + 2x + 2) \cdot e^{-x}$ is an antiderivative of f.

c) The graph of f and the x-axis are the boundaries of a region lying in the first quadrant which is open to the right.
Determine the area of this region.

4. $2 + 3 + 4 = 9$ points

Given is the sphere K_1: $x^2 + y^2 + z^2 - 2x + 10y - 18z - 118 = 0$.

a) Determine the center M_1 and the radius R_1 of the sphere K_1.

In case you are unable to solve a), continue your calculations with $M_1(1, -5, 9)$ *and* $R_1 = 15$.

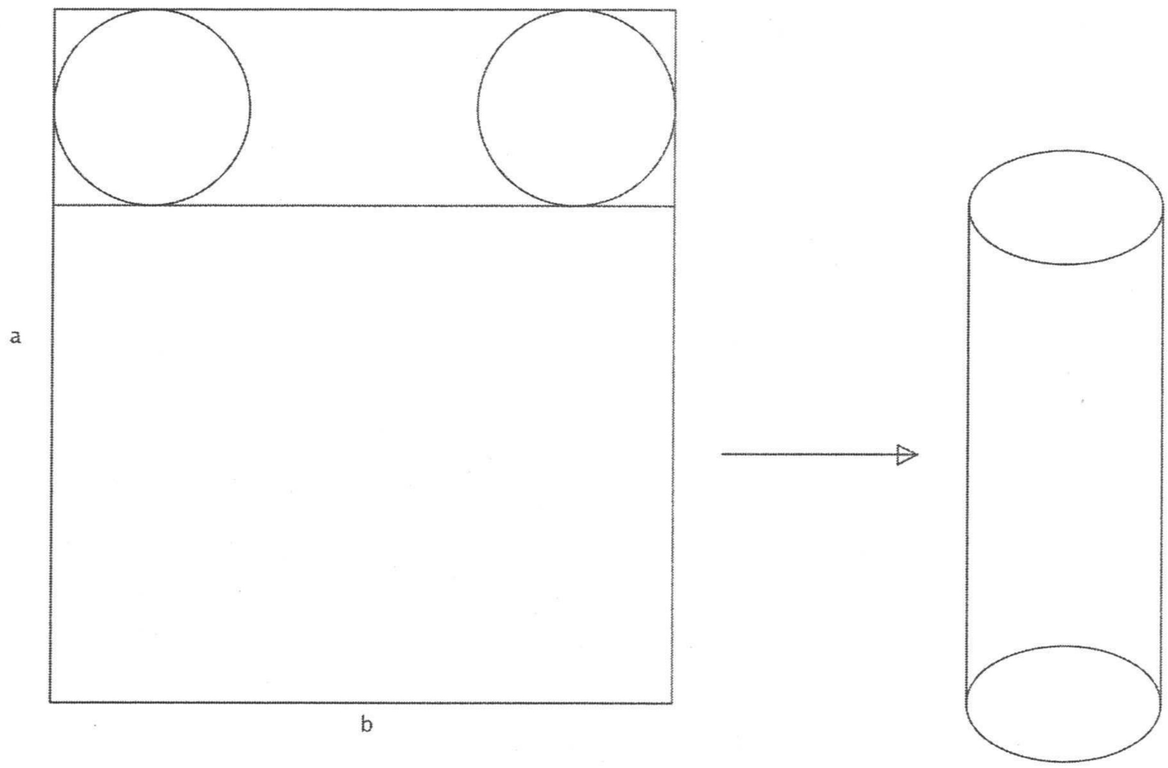
b) The sphere K_2 touches the sphere K_1 from the inside and has the center $M_2(7, -2, 3)$. Determine the point of tangency B of the two spheres (point where the two spheres touch) and the radius R_2 of the sphere K_2.

Now, the radius of the sphere K_2 is increased. A new sphere K_3 with center $M_3 = M_2$ and radius $R_3 = \sqrt{90}$ results. It intersects with the sphere K_1 in a circle.

c) Determine the center and the radius of this circle.

5.

$2 + 5 = 7$ points



A sheet of paper has side lengths a and b. A strip of paper with width $\frac{b}{\pi}$ is cut off. Two circles are cut out of this strip. The remaining sheet is coiled up to form a right circular cylinder. The two circles serve as bottom and lid.

a) Calculate the volume of the cylinder for $a = 30\text{ cm}$ and $b = 20\text{ cm}$.

b) Another rectangular sheet of paper with variable side lengths and area $A = 600\text{cm}^2$ is used in the way described above to construct a cylinder with maximal volume. Calculate the values which have to be chosen for a and b.

6.

$1 + 3 + 2 + 1 = 7$ points

Consider the complex function $f\colon\ \mathbb{C} \to \mathbb{C}$ defined by

$$f(z) = z^2 + (1-i)z - i$$

and the complex number $z_0 = -\frac{1}{2} + \frac{1}{2}i$.

a) Calculate $f(z_0)$.

b) Determine the zeros of f.

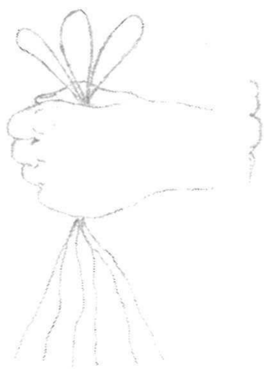
c) Determine the image of the imaginary axis. How is the resulting curve called?

d) Draw the resulting curve in the Gauss plane (1 unit = 2 cm).

Hint: Problems a) to c) can be solved independently.

7. $2+1+2+2+3 = 10$ points

Anna holds three identical pieces of string in her hand.
All six string ends are visible.
Bert has to tie the string ends together in pairs.
If the pieces of string are joined in one loop, he gets a cake from Anna.



a) Show that there are exactly 15 different possibilities to tie the string ends together in pairs (in case the order does not matter).

b) Calculate the probability that when randomly tying the string ends together in pairs three loops result. (These loops may be intertwined.)

c) Calculate the probability that when randomly tying the string ends together in pairs one loop results.

d) In case that not one loop results, several, possibly intertwined loops result. When randomly tying the string ends together in pairs, calculate the expected value for the number of loops.

Unfortunately Bert is down on his luck: He doesn't get a cake because two loops resulted. Anna gives him a second chance. She takes three new pieces of string, but now two of them are red and one is green.

e) Bert ties the string ends together in pairs and hopes to get one loop. This time he is lucky and one loop results. 7% of the male population is colour-blind and cannot tell the difference between the red and the green strings. Calculate the probability that Bert is colour-blind.