BASEL
LANDSCHAFT

BILDUNGS-, KULTUR- UND SPORTDIREKTION
GYMNASIUM OBERWIL

Final Exam 2018
Mathematics - Basic Course / Immersion

Class / Course: 4b / 4c / 4d / 4e/ 4f / 4g / 4h / 4i

Number of pages (not counting cover): 5

Content: Written final exam 2018 in mathematics, basic course

Instructions: Please start every question at the top of a **new page.**

Resources: Formelsammlung Mathematik kompakt (German or English) (Adrian Wetzel / ISBN 978-3-9523907-5-7)

Calculator TI83, TI83+, TI84+, TI84+ Silver Edition, TI84+ CE-T

Scale: The total is 78 points

The number of points is stated for each question.
You do not need to attain the full number of points for mark 6

Before you get started solving the questions, please make sure your exam paper is complete and matches the above description. If you think that something is missing, please report to the supervisors **immediately**.

i.O. [signature]

Question 1: Vector Geometry

2+2+3+3+5=15 points

Given are the plane $\mathbb{E}$ and the line g_a by

$$\mathbb{E}: \quad x + 4y + 8z + 18 = 0$$

$$g_a: \quad \vec{r} = \begin{pmatrix} 11 \\ 3 \\ 5 \end{pmatrix} + s \cdot \begin{pmatrix} a \\ 1 \\ -4 \end{pmatrix}$$

a) Let us consider the line g_a ($a \in \mathbb{R}$) where $a = \mathbf{1}$, i.e. the line

$$g_1: \quad \vec{r} = \begin{pmatrix} 11 \\ 3 \\ 5 \end{pmatrix} + s \cdot \begin{pmatrix} \mathbf{1} \\ 1 \\ -4 \end{pmatrix}$$

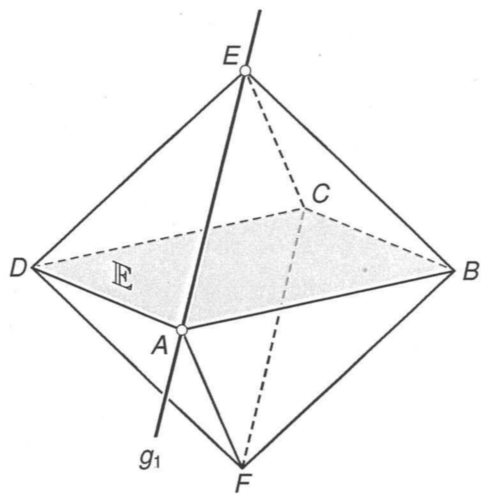
Calculate the point of intersection A of the line g_1 with the plane $\mathbb{E}$.

b) At what angle does the line g_1 intersect with the xy-plane?

c) For what value of a do the lines g_a **not** intersect with the plane $\mathbb{E}$? Describe the position of these lines with respect to the plane $\mathbb{E}$.

d) All lines g_a lie in a common plane $\mathbb{F}$. Calculate a Cartesian equation of this plane and describe the position of the plane $\mathbb{F}$ in the coordinate system.

e) The point $E(11|3|5)$ on the line g_1 and the point A which has been calculated in subproblem **a)** are vertices of the a regular octahedron (i.e. all edges have the same length) $ABCDEF$. The midsection square of the octahedron, $ABCD$, lies in the plane $\mathbb{E}$.
Choose **one** of the points B or D and calculate its coordinates.



Question 2: Rational Function

1+2+3+5+4=15 points

Given is the function f with function equation

$$f(x) = 2 - \frac{8}{x^2}$$

a) Calculate all zeros of $f(x)$.

b) Determine the equations of the horizontal and vertical asymptotes of $f(x)$.

c) Show by calculation, that $f(x)$ has neither extrema nor inflection points.

d) Calculate the equation of the tangents at the zeros of $f(x)$ and calculate the area which is bound by these tangents and the horizontal line $y = 2$.

e) In the first quadrant the y-axis, the x-axis, the graph of $f(x)$ and the line $y = 2$ enclose an area with no bounds to the right. Calculate the size of this area.

Question 3: Exponential Function

1+2+4+2+4=13 points

Given is the function f by the function equation

$$f(x) = 4x \cdot e^{-0.5x}$$

a) Calculate all zeros of the function $f(x)$.

b) Determine the behaviour of $f(x)$ for $x \to \infty$. Give an argument to support your answer!

c) Determine which kind of extremum $f(x)$ has. Calculate the coordinates **exactly**.

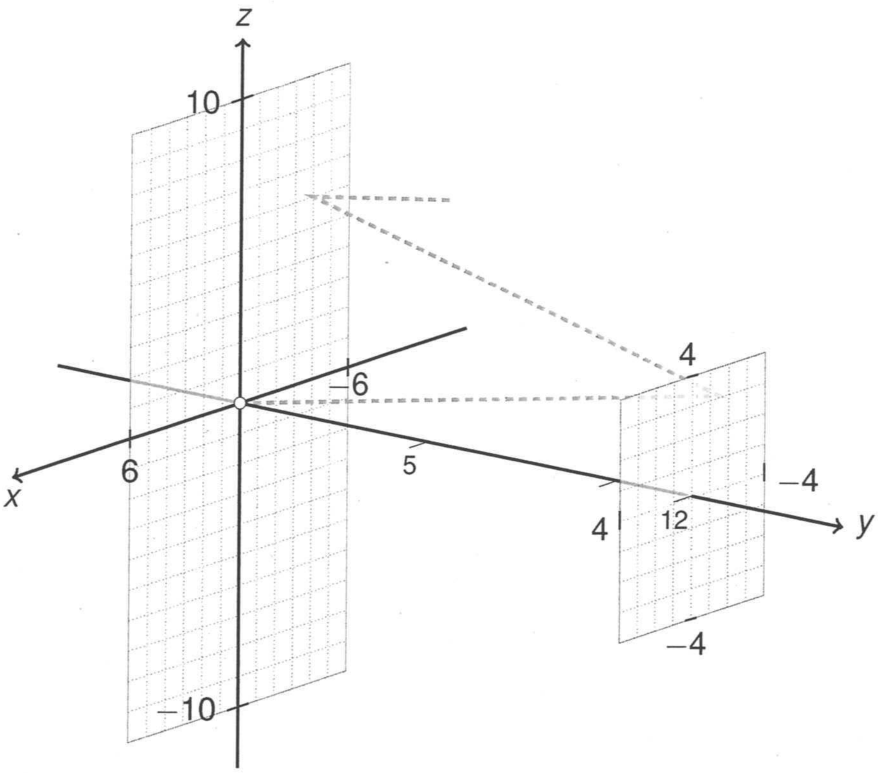
d) Prove:
G with $G(x) = -(x^2 + 2x + 2) \cdot e^{-x}$ is an antiderivative of g with $g(x) = x^2 \cdot e^{-x}$.

e) The graph of the aforementioned function f with $f(x) = 4x \cdot e^{-0.5x}$ in the interval $0 \leq x \leq 2$, the line $x = 2$ and the positive x-axis enclose an area. Calculate the **volume** which is created by revolution of this area about the x-axis.

Question 4: Parallel Mirrors

2+3+2+2+2=11 points

Two mirrors which are parallel to each other are mounted inside a Cartesian coordinate system. The larger of the mirrors lies within the xz-plane, the smaller at $y = 12$.
The sizes of the mirrors can be taken from the the drawing:



A point light source is placed at the origin $O(0|0|0)$.

a) Are there rays emanating from the origin, which will **not** strike the large mirror after reflection off the small mirror?
If so: Where would such a light ray hit the small mirror? Give an example!
If not: give an argument.

b) What is the longest way a light ray can travel once from the origin to the small mirror and back again to the large mirror?

c) A light ray ℓ leaves the light source in direction $\vec{v}_\ell = \begin{pmatrix} -0.1 \\ 1 \\ 0.2 \end{pmatrix}$

In which point does it hit the small mirror?

d) The light ray ℓ from subproblem **c)** is reflected off the small mirror. The reflected ray ℓ' appears to be emanating from the mirror image of the the light source.
Indicate a parameter equation of the line for ℓ' .

e) Another light ray g has undergone multiple reflections off the two mirrors. In the end the parameter equation of its line is: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -48 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1.2 \\ 12 \\ 1 \end{pmatrix}$
How many times was the light ray reflected off one mirror?

Question 5: Lucky-Box

2+1+2+2+3+3=13 points

The TV game show “Lucky-Box” is about 20 numbered boxes with a slip of paper inside each box that has an amount of money written on it. In addition there are five multiplication envelopes which may increase the winnings when drawn.

The following amounts and factors are hidden inside the boxes and envelopes:

Amount:	Number:
50.–	5×
100.–	5×
200.–	3×
500.–	2×
1 000.–	1×
2 000.–	1×
5 000.–	1×
20 000.–	1×
100 000.–	1×
	20×

Multiplication instruction:	Number:
“maintain”	3×
“double”	1×
“multiply by 5”	1×
	5×

Now a slip of paper indicating an amount of money and a multiplication envelope are drawn.
The winnings are calculated by multiplying or maintaining the amount of money depending on the instruction from the envelope.
Thus the first price (i.e. the maximal winnings) is

100 000.– multiply by 5 $\Rightarrow$ 500 000.–

a) In how many ways can the 20 paper slips be distributed among the 20 boxes?

What is the **probability** that…

b) … the first price is drawn?

c) … **at least** 200.– have been won?

d) What is the probability that in 25 editions of the show the winnings are more than 17 times at least 200.- ?

e) Provided you are informed that you have won 500.–, what is the probability that you have drawn a 500.– -paper slip?

f) After several years and the 1000th game of “Lucky-Box” a statistics is published. The first price (i.e. paper slip 100 000.– with envelope multiply by 5) was drawn only 3× during the whole period.

A newspaper headline reads:

Statistics uncovers scandal: “Lucky-Box” manipulated over years!

The organizers assert that the game has been conducted in a correct manner.
The players were just unlucky above average.
Take sides for one of the statements and support your decision by a calculation.

Question 6: Lobster Trap

2+5+4=11 points

A lobster trap has the shape of a half cylinder. It consists of a wire frame, which is covered by a mesh (or net).
The wire frame consists of a rectangular bottom of length ℓ, two semi-circular end covers with radius r and two additional wire parts for stabilization (cf. sketch).
The total length of the wire for the frame is denoted L. V is the total volume of the trap.

a) Express L by V and r. Show that:

$$L(r) = (2\pi + 4)r + \frac{8}{\pi r^2}V$$

b) The volume should be $V = 0.75\ \text{m}^3$.
Calculate the radius r and the length ℓ for which the least amount of wire is required!
(The examination of boundary values is not required).

c) *The solution of sub-problem* ***c)*** *can be determined using all functions of the GC without restrictions:*

The total length of the wire for the frame should be $L = 5\ \text{m}$.
For what radius r and what length ℓ does the trap exhibit the maximum surface area A (including end covers)?
Find the quantity function $A(r)$ and determine its maximum to four decimal places.