Bildungs-, Kultur- und Sportdirektion
Kanton Basel-Landschaft

GYMNASIUM OBERWIL

Finale 2016
Mathematics – Profiles A & B

Number of pages (not counting this one)	5
Content:	Written final exam in mathematics, 2016, profiles A & B
Instructions:	Please start solving **each problem at the top of a new page.**
Resources:	„Formeln, Tabellen, Begriffe" (DMK), Calculator TI-83, TI-83+, TI-84, TI-84+, TI-84+ Silver Edition English dictionary
Grading:	The total number of points is 73. The maximum number of points attainable in each problem is stated. In order to reach grade 6 you do not have to get the maximum possible number of points.

Please make sure you have a complete set of exam questions before getting started.
In case you think something is missing, inform the supervisor **immediately**.

Mathematics

Please start solving **each problem at the top of a new page.**

Duration: Four hours

Resources: „Formeln, Tabellen, Begriffe" (DMK),
Calculator TI-83, TI-83+, TI-84, TI-84+, TI-84+ Silver Edition
The regulations using the calculator of the Gymnasium Oberwil have to be followed.
English dictionary

Grading: The total number of points is 73.
The maximum number of points attainable in each problem is stated.
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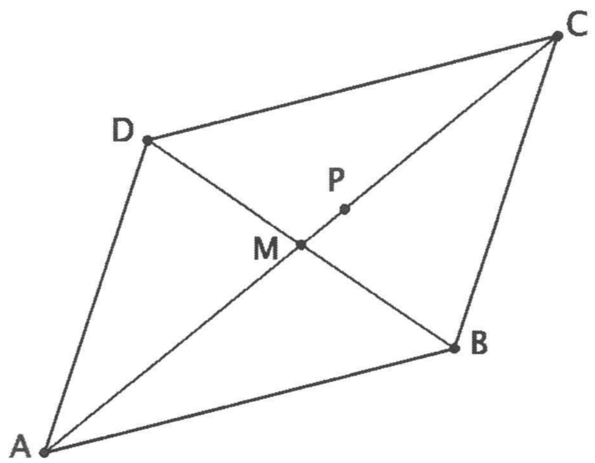
1. Given are the vertices $A(2, 7, -5)$, $B(2, 0, 9)$ and $C(11, 1, 13)$ of a parallelogram $ABCD$.

$1 + 1 + 2 + 2 + 3 + 2 = 11$ points



a) Determine the vertex D of the parallelogram.

b) Determine the point of intersection M of the two diagonals.

c) Calculate the angle $\beta = \angle ABC$.

d) The parallelogram lies in a plane ε. Determine this plane's Cartesian equation.

e) Which point P of the line passing through A and C lies closest to B?

f) The parallelogram $ABCD$ is rotated by $90°$ about the diagonal line passing through A and C. Let us call the resulting parallelogram AB^*CD^*. Calculate the coordinates of one of the two possible vertices B^*.

2. Given is the function f with

$2+3+1+3+2+2 = 13$ points

$$y = f(x) = \frac{3x}{2x^2+1}.$$

a) Determine the domain of f and the symmetry of its graph.
What happens to f for $x \to \pm\infty$?

b) Determine the maximum and minimum points of f.

c) Show that the function F with $F(x) = \frac{3}{4} \cdot \ln(2x^2+1)$ is an antiderivative of f.

d) The point $P(\frac{1}{2}, ?)$ lies on the graph of the function f.
t is the tangent to the graph of f at the point P.
Calculate the area of the region enclosed by the tangent t, the graph of f and the y-axis.

e) For every u $(u \in \mathbb{R},\ u > 0)$ the points $A(0,0)$, $B(3,0)$ and $C(u, f(u))$ are vertices of a triangle ABC. Calculate all possible values for u where the triangle's area is 1.

f) For every a $(a \in \mathbb{R},\ a \neq 0)$ the function g_a with
$g_a(x) = a \cdot f(x)$ $(x \in \mathbb{R})$ is defined.
The tangent to the graph of the function g_a at $x = 1$ is parallel to the line $y = x+2$. Find a.

3. Given is the conic section

$4+3 = 7$ points

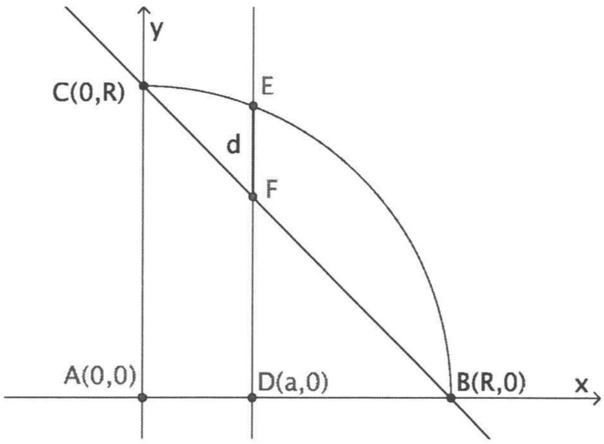
$$H:\ 2x^2 - y^2 - 4x - 4y - 4 = 0.$$

a) Show that H is a hyperbola. Determine its midpoint, its vertices (Scheitel) and the equations of its asymptotes (exact results).

b) Determine the points on the hyperbola H where the distance to the hyperbola's midpoint is 5.

4. Given is a quarter circle with center $A(0,0)$ and radius R. **2 + 5 = 7 points**

The vertical line in the point $D(a,0)$ with $0 \le a \le R$ intersects the line passing through B and C in the point F and the quarter circle in the point E (cf. diagram).



d is the distance between E and F.

a) $a = \frac{1}{3}$ and $R = 1$: Calculate d.

b) Express d by R and a. For which a does d reach its maximum?

5. The two problems a) and b) can be solved independently. **3 + 6 = 9 points**

a) Solve the following system of linear equations manually:

$$\left| \begin{array}{rcl} (1+i)z_1 - z_2 & = & 3i-3 \\ iz_1 + (1+i)z_2 & = & 0 \end{array} \right|$$

b) Given is the complex function $w = \frac{i}{z}$

b_1) Determine the fixed points of this complex function (exact results, use normal form).

b_2) Show that the image of the line $y = 1$ is a circle. Determine the center and the radius of this circle.

6. Apples can get mealy after being stored for too long. This is not visible from the outside.

3 + 3 + 3 = 9 points

a) One kind of apples contains about 20% mealy fruits after a storage of one month. Determine for apples of this kind which were stored for one month the probabilities of the following events:

a_1) 7 apples are chosen and exactly 2 of them are mealy.

a_2) 20 apples are chosen and at least 2 of them are mealy.

a_3) 100 apples are chosen. At least 15 and at most 25 of them are mealy.

b) A supermarket gets its apples from two suppliers A and B. Supplier A supplies 70% of the apples. 10% of these apples are mealy. In total 13% of all apples are mealy. Determine which supplier supplies fewer mealy apples in percents and which one supplies fewer mealy apples in numbers.

c) A fruit seller gets a load of apples with a delay. He wonders if the load contains the usual share of about 20% mealy apples or if the share of mealy apples is increased by the delay. He decides to randomly select 40 apples and to check them. He sets up the following null hypothesis: H_0: $p \leq 0.2$.

c_1) Determine the region where the null hypothesis is accepted and the region where the null hypothesis is rejected (level of significance 5%).

c_2) The check shows that 12 apples are mealy. What can the fruit seller conclude?

7. Given are the sphere K with center $M(1, 0, -2)$ and radius 15,

2 + 4 + 3 = 9 points

the plane E: $x - 2y - 2z - 41 = 0$ and the line g: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ -15 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$

a) Determine the points of intersection of the line g and the sphere K.

b) Show that the plane E intersects the sphere K. Determine the center and the radius of the circle of intersection.

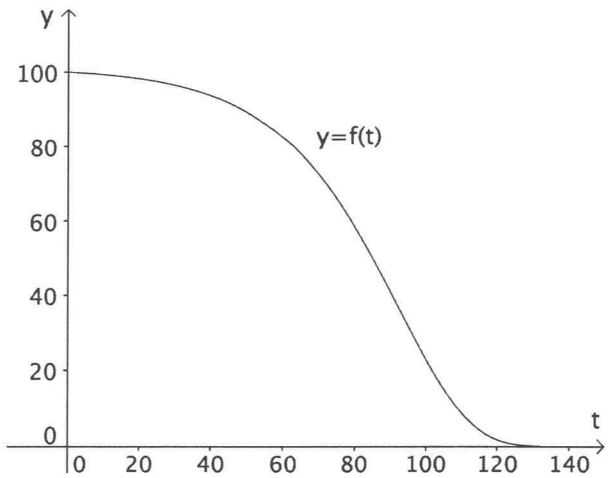
c) The sphere K touches a right circular cone with tip S from the inside at the circle found in b). Determine the tip S of this cone.

8. According to the mathematician Gompertz the lifespan of a person can be modelled in the following way: The function

1 + 2 + 2 + 3 = 8 points

$$y = f(t) = 100 \cdot e^{0.01 \cdot (1 - e^{0.05t})}$$

gives the number (in percent) of persons still alive after t years (cf. diagram to the right).



a) Calculate the percentage of persons reaching an age of at least 20 years.

b) After how many years only half of the newborn persons are still alive?

c) Calculate the momentary decrease after 20 years.

d) After how many years does the momentary decrease reach its maximum?
(You do not have to show that it is a maximum and not a minimum.)