Bildungs-, Kultur- und Sportdirektion
Kanton Basel-Landschaft

GYMNASIUM OBERWIL

Finale 2015
Mathematics – Profiles A & B

Number of pages (not counting this one) 5

Content: Written final exam in mathematics, 2015, profiles A & B

Instructions: Please start solving **each problem at the top of a new page.**

Resources: „Formeln, Tabellen, Begriffe" (DMK),
Calculator TI-83, TI-83+, TI-84, TI-84+,
TI-84+ Silver Edition
English dictionary

Grading: The total number of points is 78.
The maximum number of points attainable in each problem is stated. In order to reach grade 6 you do not have to get the maximum possible number of points.

Please make sure you have a complete set of exam questions before getting started. In case you think something is missing, inform the supervisor **immediately**.

Mathematics

Please start solving **each problem at the top of a new page.**

Duration: Four hours

Resources: „Formeln, Tabellen, Begriffe" (DMK),
Calculator TI-83, TI-83+, TI-84, TI-84+, TI-84+ Silver Edition
The regulations using the calculator of the Gymnasium Oberwil have to be followed.
English dictionary

Grading: The total number of points is 78.
The maximum number of points attainable in each problem is stated.
In order to reach grade 6 you do not have to get the maximum possible number of points.

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1. $6 + 2 + 5 + 2 = 15$ points

Given is the function $y = f(x) = -x^3 + 5x^2 - 4x$

a) Determine the zeros, the maximum and minimum points and the points of inflexion of the graph of f. Draw the graph of f.

b) Calculate the area of the region enclosed by the graph of f and the x-axis and lying in the first quadrant.

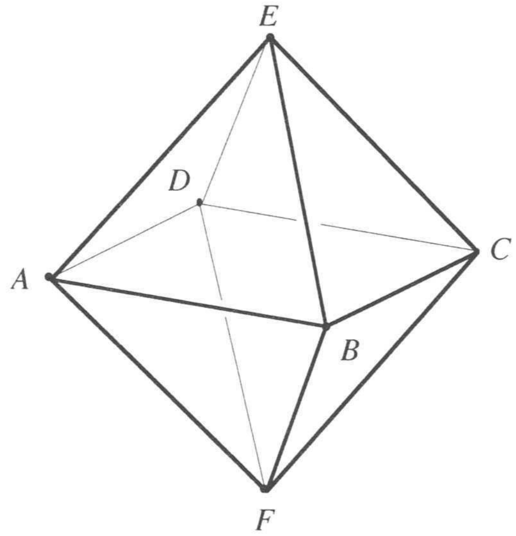
c) The point $P(x\,,\,?)$ lies on the graph of f.
The tangent t to the graph of f at the point P passes through the point $Q(-3\,,12)$. Determine all possible solutions for the point P.
Find the equation of the tangent t at one of these points.

d) Consider the generalized function $y = g(x) = -x^3 + ax^2 + bx$.
$S(2\,,\,y_S)$ is a saddle point of the graph of g. Find a, b and y_S.

2.

$4 + 4 + 2 = 10$ points

Given are the tips $E(5, 1, 0)$ and $F(1, 5, 2)$ of a right dipyramid (gerade Doppelpyramide) with quadratic base $ABCD$ (cf. diagram to the right).



a) The vertex A lies on the line

$$g: \vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}.$$

Determine A.

In case you were unable to solve a) continue with $A^*(5, 4, 3)$.

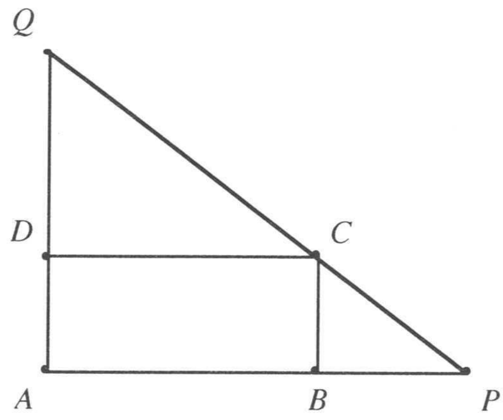
b) Determine the vertices B, C and D.

c) (can also be solved without b)) Calculate the volume of the dipyramid.

3.

$2 + 5 = 7$ points

$ABCD$ is a rectangle with side lengths $a = \overline{AB} = 9\,\text{cm}$ and $b = \overline{BC} = 4\,\text{cm}$. APQ is a right-angled triangle (cf. diagram to the right).



a) Calculate the sum of the lengths of the triangle's legs AP and AQ for $\overline{BP} = 5\,\text{cm}$.

b) Consider the sum of the lengths of the triangle's legs AP and AQ. Detemine the minimum of this sum. Show that your result is a minimum.

4.

$10 + 4 = 14$ points

a) Caused by the avian influenza („bird flu“) the demand for chicken meat decreased during a certain time span. After a while it recovered.
For one company the demand t days after the start of the avian influenza was

$$f(t) = 20 - 0.4 \cdot t \cdot e^{-0.01t} \quad \text{(tons per day, } t \geq 0\text{)}$$

i) How many tons per day did the company normally sell?

ii) Determine $\lim_{t \to \infty} f(t)$. Interpret the meaning of this limit.

iii) After how many days did the demand reach its minimum?
How many tons were sold on this day?

iv) Sketch the graph of the function f for $0 \leq t \leq 500$

v) Mark the point in your sketch where the increase of the demand is as large as possible. What is the mathematical term for this point on the curve?

vi) Show that $F(t) = 20t + 40 \cdot t \cdot e^{-0.01t} + 4000 \cdot e^{-0.01t}$ is an antiderivative of f.

vii) By how many tons was the company's sale reduced in the first 400 days after the start of the avian influenza?

b) The demand for chicken's eggs decreased during this time as well.
t days after the start of the avian influenza the demand was

$$g(t) = a - b \cdot t \cdot e^{-kt} \quad \text{(thousand eggs per day, } t \geq 0\text{)}$$

Normally, the company sold 150 thousand chicken's eggs per day.
At the time $t = 0$ the demand decreased by 6 thousand eggs per day.
After 50 days the demand reached its minimum.
Find a, b and k.

5. **2 + 2 + 1 + 2 + 4 = 11 points**

Given: line g: $\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 9 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

sphere K: $x^2 + y^2 + z^2 - 2x + 14y - 6z - 22 = 0$

a) Determine the center and the radius of the sphere K.

b) Detemine the points of intersection A and B of the line g and the sphere K.

In case you were unable to solve b), continue with the points $A^*(0, -15, -1)$ *and* $B^*(8, -11, 7)$ *of the sphere.*

c) Determine the center and the radius of the largest circle lying on the surface of the sphere and passing through A and B.

d) Detemine the center and the radius of the smallest circle lying on the surface of the sphere and passing through A and B.

e) Detemine the center and the radius of a circle lying on the surface of the sphere, passing through the points A and B and lying in a plane normal to E: $3x + 2y + 5z = 0$.

6. **4 + 2 + 1 + 1 = 8 points**

a) Solve the equation $z^3 = 8i$. Write the solutions z_1, z_2 und z_3 using standard form and polar form (exact results). Draw the three points in the Gauss plane.

b) Show that the points form an equilateral triangle and calculate its perimeter.

c) Show $z_1 \cdot z_2 \cdot z_3 = 8i$

d) Show $z_1 + z_2 + z_3 = 0$

7. $1 + 2 + 3 + 3 + 4 = 13$ points

In Switzerland the probability for the birth of twins is 1.9%.
The probability for these twins to be identical (eineiig) is 20%.
Identical twins are the same sex.
The probability for one twin to be female is 48.5%.

a) Yesterday there were two deliveries (Entbindungen) in a Swiss hospital.
What is the probability that in boths cases twins were born?

b) What is the minimum of deliveries a hospital has to register if the probability that twins are born at least once is larger than 95%?

c) Calculate the probability that a randomly selected delivery is a birth of twins of different sexes.

d) At a randomly selected birth of twins two girls are born. What is the probability that they are identical?

In a certain year there were 1478 births of twins in Switzerland.
The statistics showed the following numbers:

Two girls:	415 births
One girl, one boy:	631 births
Two boys:	432 births

Reminder:
The probability for Swiss twins to be identical is 20%.
The probability for one twin to be female is 48.5%.

e) i) Taking the probabilities mentioned above into account:
What is the number of births to be expected for the case „one girl, one boy"?

ii) Is it possible to conclude from this result that the probability for a twin birth of one girl and one boy was significantly changed?
(Two-sided test, probability of error 4.5%)