Question 1: Discussion of a function

16 points

The function f is given by its equation

$$f(x) = \frac{1}{4}(x+2)^2(x-3) = \frac{1}{4}x^3 + \frac{1}{4}x^2 - 2x - 3$$

a) Calculate all zeros of the graph of f and the coordinates of all stationary points and points of inflexion.

b) Calculate the angle α at which the graph of f intersects the ***y* axis**.

Also, a linear function g ist given by its equation

$$g(x) = x - 3$$

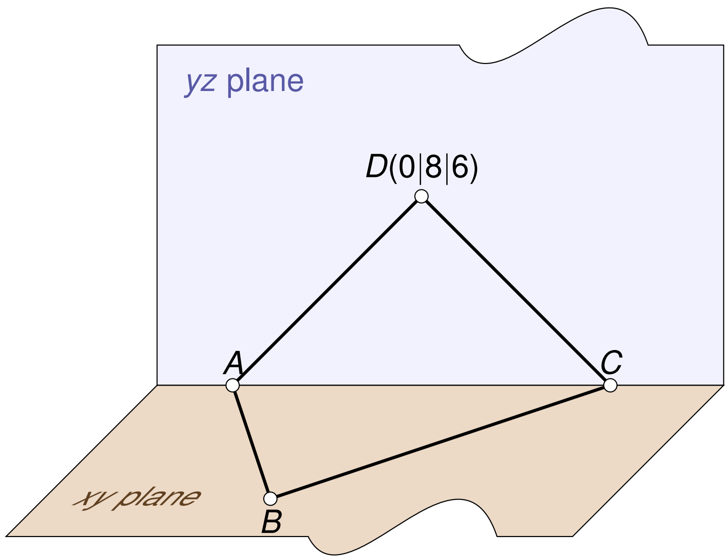
c) Calculate the points of intersection of the graphs of f and g.
Hint: you should use the factorised form of the function equation.

d) Calculate the area of the finite region bordered by the graphs of f and g.
(In case you weren't able to solve ***c)****, you may use the graphing features of your calculator to find the points of intersection.)*

Question 2: Folded quadrilateral

7 points

The corners of a folded quadrilateral lie in the xy and yz planes of a coordinate system:



a) Find the coordinates of A and C, such that the triangle CDA is both isosceles and right-angled. The right angle should be at D.

b) Find the coordinates of B, such that triangle ABC is isosceles and right-angled, too, with right angle at B.

(In case you weren't able to solve ***a)*** *and* ***b)****, use the following coordinates instead:* $A(0|6|0)$, $C(0|26|0)$, $B(6|8|0)$. *The triangles will still be right-angled, but not isosceles.)*

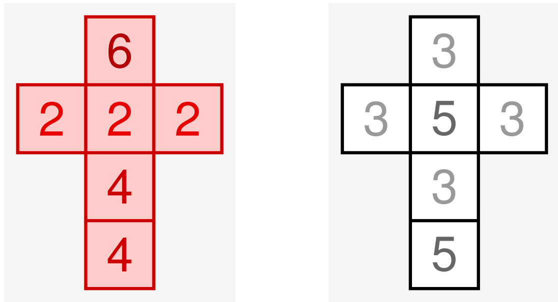
c) Calculate the angle $\measuredangle DCB$ (the apex of which is at C).

d) The points A und B define a straight line g. Calculate the coordinates of the point S at which g passes through the xz plane.

Question 3: Coloured dice

14 points

A box contains 20 red dice and 10 white dice the nets of which are shown in the figure.



You draw one dice, roll it, write down the colour and score, and put the dice back into the box. This procedure is carried out **three times**.

Calculate the probabilities of the following events:

a) All three of the dice are the same colour.

b) If you multiply the three scores, you get an even number.

c) In every roll, the number on the upper face is the same as the number on the bottom face.

Now **one** of the same 30 dice was selected at random and that dice was rolled **twice**. The two scores were found to add up to 6.

d) What is the probability that a *red* dice was used?

Bianca and Ruby play the following game: Bianca rolls a white dice and Ruby rolls a red dice. Whoever attains the higher score will be paid one franc for every dot in her own score by the other player.

e) What average gain or loss per round should Bianca expect?

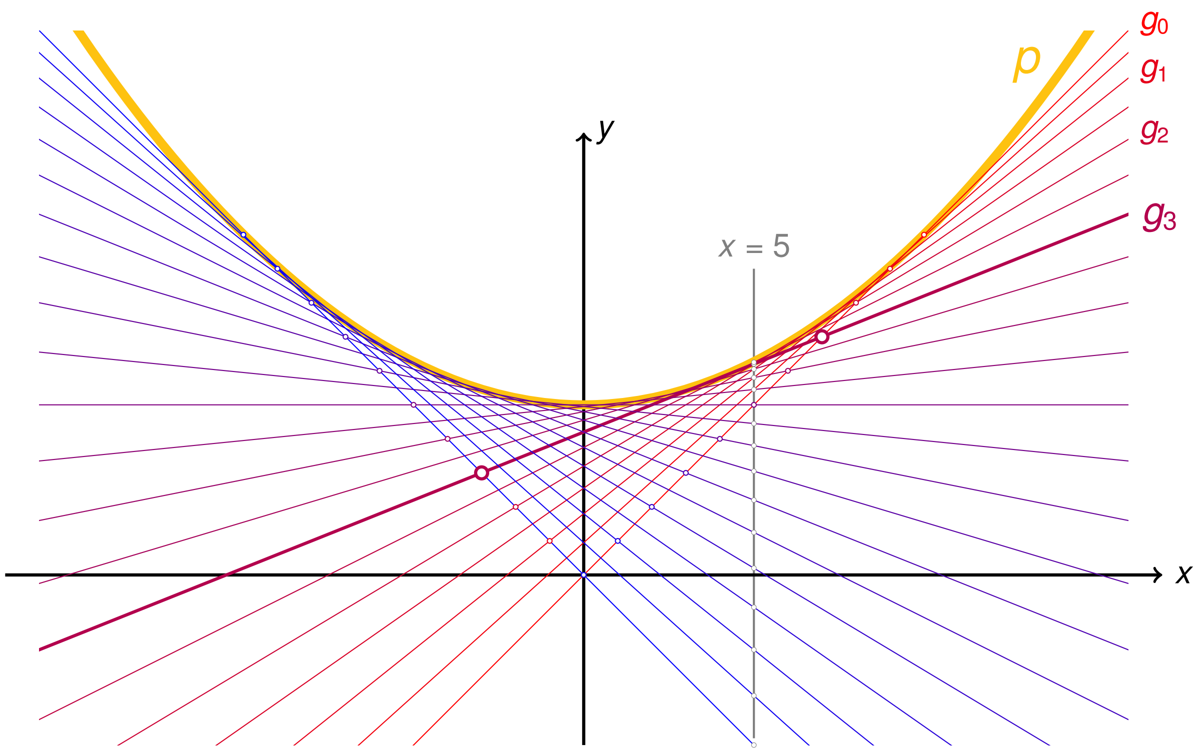
Question 4: Envelope (“Hüllkurve”)

12 points

The following figure shows some lines $g_0, g_1, g_2, g_3, \ldots$, where each g_t passes through the points

$$P(-t|t) \quad \text{and} \quad Q(10-t|10-t).$$

The parameter $t \in \mathbb{R}$ lies in the range $0 \leq t \leq 10$.



When we draw the family of all lines so defined, these lines wrap around some curve p. The curve is known as the **envelope** of the family of lines.

a) State the function equation of the line g_3.

b) Show that the line g_t has function equation

$$g_t(x) = (1 - 0.2 \cdot t)\, x \; + 2t - 0.2t^2$$

c) We shall now consider the vertical line with equation $x = 5$ (see figure). It is crossed by all the lines g_t.
Calculate t such that the corresponding point of intersection has the largest possible y coordinate.

Each of the lines contributes one point to the **envelope** p: It is the point which—compared to the points on all other lines at that place—has got the largest y coordinate. You made use of this idea in part **c)** to determine **one** point on the envelope.
The envelope is a parabola with its vertex on the y axis, which means that its function equation is of the form

$$p(x) = ax^2 + c$$

d) Calculate the function equation of the envelope p.

Question 5: Triangle

12 points

Given the triangle ABC with

$$A(3|6|16), \quad B(9|12|-8), \quad C(-15|6|-2)$$

along with the point $P(-8|4|15)$.

a) Show that the triangle ABC is equilateral.

b) Show that the point P lies in the same plane as the triangle ABC.

c) Does the point P lie inside or outside the triangle ABC? Your answer should be based on calculation.

d) The points A, B and C lie on a sphere the centre M of which is located in the xy plane. Calculate the coordinates of M.
Hint: Figure out where all the points lie that have equal distances from A, B, and C.

Question 6: How many copies?

4 points

100 employees of a company have signed up for a training course. All of them were mailed the course handout in advance, and were asked to bring the handout to the course.
Experience tells that some of them will leave their handout at home: this happens to each individual participant with probability $p = 0.2$

a) The course instructor keeps 25 spare copies of the handout ready. Calculate the probability that these spare copies will be enough.

b) At the beginning of the course, only 11 have left their handout at home.
The secretary says: "Our folks have gotten better at remembering their handouts this year. Perhaps it helped that I used a red marker in the invitation letter."
The course instructor says: "They're just as sloppy as always, we just happened to be lucky this year."
Take sides with one of these statements. Your reasoning should be based on a calculation.

Question 7: Exponential function

11 points

The function f is given by its equation

$$f(x) = c \cdot a^x \quad \text{where } a, c \in \mathbb{R} \text{ and } a > 0$$

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a) Calculate the parameters a and c such that the graph of f passes through the points $P(4|\frac{1}{4})$ and $Q(-1|8)$.

For all of the following parts, use the values of a and c found in part **a)**.
(In case you weren't able to solve ***a)****, continue using $f(x) = 72 \cdot \left(\frac{1}{3}\right)^x$ instead.)*

Another function, g, is given by its equation

$$g(x) = 0.5 \cdot 4^x$$

b) Calculate the point of intersection $S(x_s|y_s)$ of the graphs of f and g.

c) State the equation of the tangent to the graph of g, touching the graph at the point of intersection S. The values of the slope and y intercept should be given in **exact** form.
(In case you weren't able to solve ***b)****, you may use the graphing features of your calculator to find the point of intersection.)*

d) The graph of g over the interval $[-2; 0]$ is now rotated about the x axis.

Calculate the volume of the resulting solid with an accuracy of two decimal places.