

# Mathematics

Please **start solving each problem at the top of a new page!**

**Duration:** 4 hours

**Resources:** "Fundamentum", English dictionary, Calculatort TI-nSpire (**without** CAS)

**Grading:** The maximum number of points attainable in each problem is stated.  
In order to reach grade 6, you do **not** have to get the maximum possible number of points.

1) The points  $A(3 / -3 / 0)$ ,  $B(2 / 0 / 8)$ ,  $C(2 / -2 / 4)$  as well as the line

10 points
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$$g_a : \vec{r} = \begin{pmatrix} -3 \\ 3 \\ 0 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ -1 \\ a \end{pmatrix} \text{ are given.}$$

- Set  $a = 1$ . What is the angle of intersection between  $g_1$  and the  $xy$ -plane?
- Find the cartesian equation of the plane  $\mathbb{E}$  that is defined by the points  $A$ ,  $B$  and  $C$ .
- Determine the axes-intersection points  $X$ ,  $Y$  and  $Z$  of the plane  $\mathbb{E}$ . These points are the vertices of a triangle. Show this triangle in a coordinate system.
- For what value of  $a$  is  $g_a$  perpendicular to  $\mathbb{E}$ ? Find the coordinates of the point of intersection for this case.
- The origin  $O$  of the coordinate system and the points  $X$ ,  $Y$  and  $Z$  are the vertices of a pyramid. For what value of  $z \in \mathbb{R}$  does the point  $P(1 / -4 / z)$  lie inside the pyramid  $OXYZ$ ?



2) For the construction of a tent in the form of a right cone poles of length 5m are available. These poles are used to support the tent's tarpaulin ("Zeltplane") which forms the cone's lateral surface.

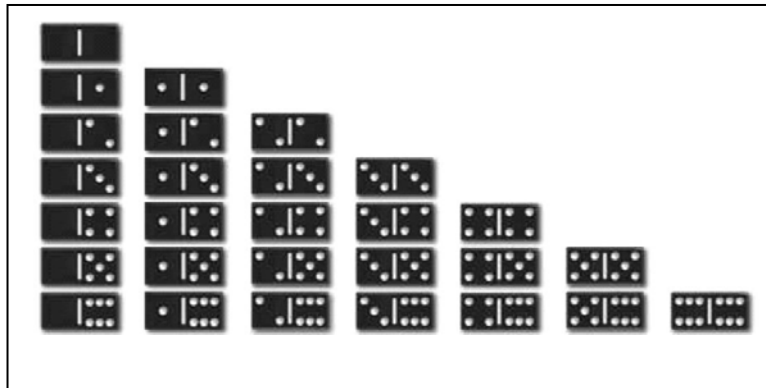
7 points
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- The height of the tent is 4m.
  - Calculate the content of the circular base area of the tent.
  - Out to what distance from the tent's "center" will a person that is 1.8m tall be able to stand upright?
- Find the height of the tent for which the tent's volume reaches a maximum.

- 3) A bag contains 28 Domino stones (see below). A stone with two equal number of points on it is called a "doublet". We denote this number of points as the "size" of the doublet.

12 points

Therefore,  is a doublet of size 5 and  is a doublet of size 0.



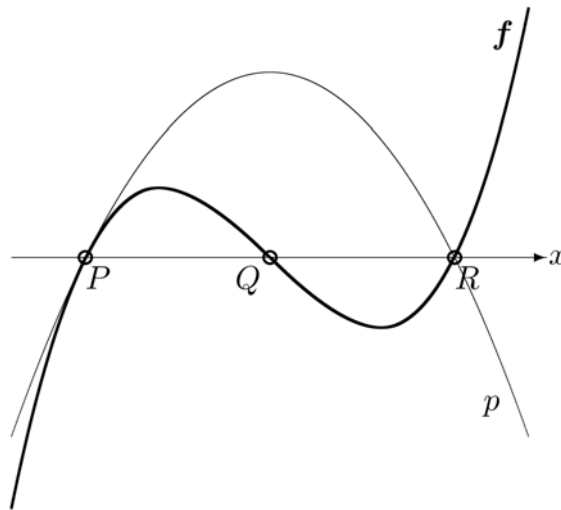
- a) Somebody draws two stones randomly and at once from the bag. Calculate the probability for the following results:
- Both stones are doublets.
  - At least once the number of points 5 shows up.
  - The sum of all four number of points is three.
  - Four different number of points show up.

All stones are now returned to the bag.

- b) You draw stones **without returning them to the bag** until you get a doublet for the first time. If the number of drawn stones does **not** surpass the size of this doublet, you receive 1 Euro for each stone drawn (the doublet included). What stake in Euros would make this a fair game?
- c) The 28 Domino stones fall to the ground. 18 of the stones land face up. What is the probability for this or an even more extremal outcome (even more stones landing face up), if we assume that a single Domino stone will land face up with a probability of 50%?

- 4) The graph of the function  $y = f(x) = x^3 - x$  (shown in bold type in the sketch below) intersects the x-axis in three points P, Q and R.

12 points
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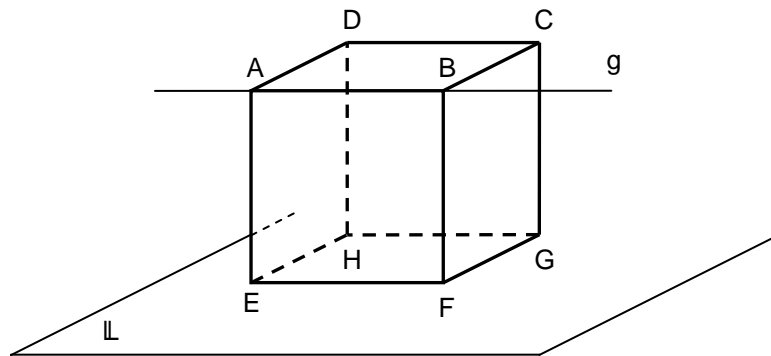
- Determine the coordinates of P, Q and R and draw the y-axis in the sketch above.
- Give the equation of the parabola  $p$  that **touches** the graph of  $f$  in P and intersects this graph at R (see sketch above).  
(In case you were not able to solve problem b), you may continue using the (incorrect) result  $y = p(x) = 1.5 - 1.5x^2$  !)
- Calculate the content of the area G inclosed by the two graphs.
- For what value of  $x$  is the vertical distance between the two graphs bounding G the largest?
- Let  $t$  be the tangent line to the graph of  $f$  which touches the graph between P and Q and has a slope of  $-\frac{1}{4}$ . Give an equation for  $t$ .
- What percentage of G lies **above** the tangent  $t$  found in problem e)?

- 5) The line  $g$  passing through  $A(3 / 2 / 0)$  and  $P(5 / 0 / 1)$  as well as the line

10 points

$$h: \vec{r} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + s \cdot \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix} \text{ are given.}$$

- Show that  $g$  and  $h$  are perpendicular to each other and do **not** intersect.
- All the lines parallel to  $g$  that intersect  $h$  define a plane  $\mathbb{L}$ . Give the cartesian equation of  $\mathbb{L}$  and calculate the distance of the line  $g$  from  $\mathbb{L}$ .
- $A$  is a vertex of a cube  $ABCDEFGH$  (see sketch) whose leg  $AB$  lies on  $g$  and whose surface  $EFGH$  is part of  $\mathbb{L}$ . Determine the coordinates of the vertices  $E$ ,  $B$  and  $H$  for one of the possible solutions.



- 6) The functions  $y = f(x) = x^2 \cdot e^{-x}$  and  $y = g(x) = e^{-x}$  are given.

12 points

- For the function  $f$ , give the zeros and the extrema (without proof of the type) and sketch the graph.
- Determine the points of intersection of  $f$  and  $g$ . Find the angle of intersection in one of the points of intersection.
- An anti-derivative of  $f$  has the form  $F(x) = (a \cdot x^2 + b \cdot x + c) \cdot e^{-x}$ . Determine the values for  $a$ ,  $b$  and  $c$ .
- Find the content of the area which extends indefinitely to the right and is bounded by the graphs of  $f$  and  $g$  as well as by the  $x$ -axis.